Applications of the Dynamic Circuit Theory to Maglev Suspension Systems

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Abstract—This paper discusses the applications of dynamic circuit theory to electrodynamic suspension EDS systems. In particular, the paper focuses on the loop-shaped coil and the figure-eight-shaped null-flux coil suspension systems. Mathematical models, including very general force expressions that can be used for the development of computer codes, are provided for each of these suspension systems. General applications and advantages of the dynamic circuit model are summarized. The paper emphasizes the transient and dynamic analysis and computer simulation of maglev systems. In general, the method discussed here can be applied to many EDS maglev design concepts. It is also suited for the computation of the performance of maglev propulsion systems. Numerical examples are presented in the paper to demonstrate the capability of the model.

I. INTRODUCTION

A MAGLEV system uses magnetic forces to perform the functions of levitation, guidance, and propulsion of a vehicle. The computations of these magnetic forces may differ slightly from those in conventional electrical machines for the following reasons. First, knowledge of three-dimensional time- and space-dependent magnetic forces are required to treat the basic six degrees of freedom of a maglev vehicle. Second, space harmonics, which result from the end-effects and the discontinuous distribution of the magnets aboard the vehicle, play a much more important role in the performance of a maglev vehicle. Thus, the performance analysis based on a fundamental traveling wave used in most conventional machines is inadequate. Third, time-dependence of the magnetics forces (i.e., the transient as well as time-averaged characteristics of the forces) are important because they influence the noise and vibration of components, heat loads on superconductors, and vehicle motions and ride quality.

Several approaches are widely used for the computation of magnetic forces in maglev systems. The finite-element method is one of the more powerful numerical techniques for solving Maxwell’s field equations. For given boundary conditions and specified system geometry, one is able to obtain sufficient information for a system using two- and three-dimensional finite-element computer codes. However, when a system involves relative motions with space and time dependences, the finite-element method becomes difficult because a great amount of computing time is required to obtain the force-speed, or force-time, characteristics. In addition, most commercially available finite-element computer codes do not include the problems associated with moving conductors [1]. Fourier transformation, or harmonics analysis, in combination with numerical techniques, is another powerful method that is applied to maglev problems, such as determining the lift and drag forces in a continuous sheet guideway. The method, however, is usually limited to a two-dimensional steady-state analysis with an assumption of infinite guideway width.

The dynamic circuit theory, also called general machinery theory or mesh-matrix method, is a suitable approach for maglev applications. It can overcome some of the limitations mentioned above and take both end-effects and edge-effects of the electromagnetic system into consideration. The dynamic circuit theory treats an electrodynamic system in terms of space- and time-dependent circuit parameters governed by a set of differential equations in matrix form. When plate or sheet conductors are considered, the method divides the conductors into many zones, each of which carries a different current. The circuit parameters for every conducting zone are then determined, and the system of equations is formed. Once the system of equations is solved for the current distribution, the forces acting between the electrodynamic system components can be calculated in a straightforward manner. Therefore, the performance of the system can be investigated. Since the equations are usually solved for the currents in the time domain, the method is well suited for transient and dynamic analysis and for the computer simulation of electrodynamic systems, such as maglev trains, electromagnetic launchers, and other electrical machines.

Analyses of rotating electrical machines based on the general theory of electrical machines are discussed by Morgan [2]. Analyses of linear machines using the mesh-matrix method have been reported [3]-[4]. The dynamic circuit theory used for electromagnetic launcher analysis and simulation was discussed in several publications [5]-[7], where the transient and the dynamic performance are emphasized. In launcher analyses, a relatively short time period (a fraction of second) is considered because of the hypervelocity of the projectile. In addition, a capacitor
bank or a pulsed generator is used as the power source for electromagnetic launchers. The dynamic circuit theory used for the computation of a continuous-sheet suspension in a maglev system was discussed by a Canadian maglev group [8]–[9]. In their model, the dynamic circuit theory was combined with a harmonic analysis. The superconducting magnets aboard the vehicle was replaced by a current sheet that was expressed in terms of a Fourier series. A d-q transformation, which is usually used in rotating machine theory to transform a rotating machine into a stationary primitive machine, was applied to the direction of motion for all harmonics. The performance of the continuous-sheet guideway was determined on the basis of the circuit solutions in combination with the superposition theorem.

Although the dynamic circuit theory has been discussed with respect to certain applications in other papers as mentioned above, applications of the theory to various maglev suspension and propulsion systems involving nonsteady-state forces or discrete coils (as opposed to continuous-sheet conductors) have not been thoroughly discussed. In particular the use of dynamic circuit theory to simulate the performance of a complete maglev system with respect to certain applications in other papers as mentioned above, applications of the theory to various nonsteady-state forces or discrete coils (as opposed to maglev suspension and propulsion systems involving continuous-sheet conductors) have not been thoroughly reported previously. In this paper, we apply the dynamic circuit theory to several electrodynamic suspension systems, including a loop-shaped coil suspension and a figure-eight-shaped null-flux suspension, and emphasize a direct solution and direct computation of forces without using Fourier and d-q transformations or postcomputation processing. The paper provides mathematical models for various suspension systems and exposes their similarities and differences on the basis of dynamic circuit theory. An attempt is also made to provide some physical insight into the relationships between system parameters and the nature of the resulting forces. This insight, in turn, provides some guidance for the design of discrete-coil maglev systems. The mathematical models can be used for the development of the computer codes that are necessary for the design, analysis, and simulation of large-scale maglev systems. Indeed, work is currently in progress at Argonne on the development of maglev-system-simulating computer codes to account for both the electrodynamics and mechanical dynamics of vehicles interacting with guideways.

The remainder of the paper discusses several topics: the dynamic circuit theory, application of the theory to loop-shaped coil suspensions, application of theory to the figure-eight-shaped null-flux coil suspension systems, the transient and dynamic performance of the null-flux coil suspension system, and conclusions about the theory and its applications.

II. GENERAL MODEL

A. Energy Conservation and Forces in a Maglev System

A maglev system can be represented by the dynamic circuit model in which the system energy, power, and forces, as well as other quantities, are expressed in terms of their equivalent circuit parameters. In general, those circuit parameters are functions of time and space. Thus, the dynamic and transient performance of a maglev system can be determined on the basis of the solution of the dynamic circuit model.

In general, we may consider a maglev system in which m vehicle coils or conductors interact with n guideway coils or conductor to produce levitation, guidance, and propulsion forces. All these coils are assumed to be connected to individual power sources. Thus, the superconducting coils aboard the vehicle can be represented by letting the terminal voltages and resistances of the vehicle coils vanish, the passive guideway conductors can be represented by letting terminal voltages vanish, and the propulsion system can be represented by connecting a poly-phase power source to the guideway stator coils. Assuming [i] and [e] to be column (m + n) matrices made up of the individual currents and voltages associated with the vehicle and guideway coils or conductors, respectively; [L] to be a square (m + n) × (m + n) matrix, each element of which represents either the self-inductance of a vehicle or guideway coil or the mutual inductance between a vehicle coil and a guideway coil; and [R] to be a diagonal (m + n elements) matrix made up of the individual vehicle coil and guideway coil resistances, we can write the system voltage equations in matrix form on the basis of Kirchoff's voltage law

\[ [e] = [R][i] + \frac{d}{dt}[[L][i]]. \]  

In general, a maglev vehicle (or a vehicle-borne coil) undergoes motions of three dimensions. Letting \( v_x, v_y, \) and \( v_z \) be the velocities of the vehicle in the x, y, and z directions, respectively, we can rewrite (1) in terms of speed voltages (i.e., voltages induced by relative motions):

\[ [e] = [R][i] + v_x[G_x][i] + v_y[G_y][i] + v_z[G_z][i] \]

\[ + v_x[G_x][i] + v_y[G_y][i] + v_z[G_z][i], \]

where \( [G_x] = \frac{\partial[L]}{\partial x}, [G_y] = \frac{\partial[L]}{\partial y}, \) and \( [G_z] = \frac{\partial[L]}{\partial z}. \) The total time-dependent electrical power input, \( P \to \) a maglev system is

\[ P = [i]^T[e] = [i]^T[R][i] + [i]^T[L] \frac{d}{dt} [i] \]

\[ + v_x[i]^T[G_x][i] + v_y[i]^T[G_y][i] + v_z[i]^T[G_z][i]. \]

Equation (3) where the superscript \( T \) stands for the matrix transpose. Rearranging the right-hand side of (3), one obtains:

\[ P = [i]^T[R][i] + \frac{d}{dt} \left\{ \frac{1}{2}[i]^T[L][i] \right\} + \frac{1}{2}v_x[i]^T[G_x][i] \]

\[ + \frac{1}{2}v_y[i]^T[G_y][i] + \frac{1}{2}v_z[i]^T[G_z][i] \]

Equation (4) shows the power balance of a maglev sys-
tem. We note in (4) that the term on the left represents the total electrical power input to the system, which may include the power from a stationary power system and the power from the batteries aboard the vehicle. The first term on the righthand side represents the dissipated power of the system, which may include the power losses both in the guideway coils or conductors and in the vehicle coils if superconducting magnets are not used aboard the vehicle. Even if superconducting coils are used, some power losses may arise, and they can be represented by the first term. The second term represents the time rate of change of the magnetic energy stored in the system, and the last three terms on the right-hand side represent the converted mechanical power due to the three-dimensional motion of the vehicle. Finally, the three force components acting on the vehicle, \( f_x, f_y, \) and \( f_z \), can be obtained from (4) by dividing the terms of the converted mechanical power by their corresponding velocity components, \( v_x, v_y, \) and \( v_z \), respectively.

\[
f_x = \frac{1}{2}[i]^{T}[G_v][i] \\
f_y = \frac{1}{2}[i]^{T}[G_g][i], \quad (5) \\
f_z = \frac{1}{2}[i]^{T}[G_v][i]. \quad (6)
\]

Equation (9) shows that the longitudinal time-dependent magnetic force in a maglev system consists of two terms: the dissipative term and the nondissipative term. In time-averaged calculations, the first term of (9) yields a well-known magnetic drag, and the second term vanishes.

### B. Transformation for the Coil Connections

A maglev system usually involves many coils that may logically be connected in several different groups to perform different functions as levitation, guidance, and propulsion. For instance, the figure-eight-shaped null-flux coil guideway can be viewed as two loop-shaped coils connected in reverse direction. Propulsion coils, in general, are connected into three groups to form three-phase armature windings. Other maglev systems are expected to have even more complicated coil connections to perform an integrated maglev function. The dynamic circuit model can be applied to many maglev systems, if the transformation of the coil connections is considered.

General transformations for solving electrical machine problems were thoroughly discussed by Morgan [2]. The transformation for the coil connections is particularly useful for the maglev simulation and analysis on the basis of the dynamic circuit model. Because the vehicle and guideway coils are usually connected in different configurations that may need different transformations, it is necessary to partition all the matrices by rows or columns to form submatrices. Thus, the previously defined current and voltage matrices expressed in terms of submatrices are

\[
[i] = \begin{bmatrix} I_v \\ I_g \end{bmatrix}, \quad (10)
\]

\[
[e] = \begin{bmatrix} E_v \\ E_g \end{bmatrix}, \quad (11)
\]

where the bold letter \( I_v, I_g \) and \( E_v, E_g \) are the current and voltage submatrices of the vehicle coils and guideway coils, respectively. The subscripts \( v \) and \( g \) stand for the vehicle and guideway, respectively. The inductance matrix becomes

\[
[L] = \begin{bmatrix} L_v & L_{vg} \\ L_{vg} & L_g \end{bmatrix}, \quad (12)
\]
where \( L_v \) and \( L_g \) are the inductance submatrices of the vehicle coils and guideway coils, respectively. Thus \( L_{vg} = L_g \) are the submatrices that represent the coupling between the vehicle coils and guideway coils. They are the most important part of the system because all magnetic forces are generated from this coupling. Similarly, the resistance matrix in the system may be partitioned into submatrices \( R_v \) and \( R_g \):

\[
[R] = \begin{bmatrix}
R_v & 0 \\
0 & R_g
\end{bmatrix},
\]

The \( R_v \) becomes a zero submatrix when superconducting coils assumed to have no losses are used aboard the vehicle. One can define a transformation matrix \([T]\) as

\[
[T] = \begin{bmatrix}
T_v & 0 \\
0 & T_g
\end{bmatrix}
\]

where \( T_v \) and \( T_g \) are the transformation submatrices for the vehicle coils and the guideway coils, respectively, which depend on the connection of the coils. The term \( T_y \) may be a unit submatrix if the transformation is only applied to the guideway coils. If the prime quantities are introduced as a new system after transformation, then, on the basis of power invariance, one can obtain for the current \( I \)

\[
[I] = [T][I]',
\]

and for the voltage \( V \)

\[
[V]' = [T]^T[V].
\]

The inductance matrix and its derivative matrix of the new system are

\[
[L]' = [T]^T[L][T],
\]

\[
[G]' = [T]^T[G][T],
\]

and

\[
[R]' = [T]^T[R][T].
\]

By substituting the prime quantities in (15) to (19) into (1) into (9), one can obtain the power balance and force equations for the new system. Typical examples for the use of the transformation will be discussed in the following sections.

III. APPLICATION OF THE THEORY TO THE LOOP-SHAPED COIL SUSPENSION

Considerable attention has been given to suspension schemes in which the superconducting coils are levitated above a loop-shaped coil guideway, as shown in Fig. 1. It has been suggested that the coil guideway, because of its relatively low magnetic drag, is superior to the continuous-sheet guideway [10]. The loop-shaped coil guideway, however, produces force pulsations that do not arise in the continuous-sheet guideway (except for those caused by discontinuities associated with thermal expansion joints). A steady-state analysis of the loop-shaped coil guideway has been discussed elsewhere [11], on the basis of the Fourier transformation method in combination with steady-state circuit analysis. Takano et al treated the case where end effects occur using the dynamic circuit method in combination with perturbation theory [12]. The dynamic circuit model is a generalized approach that is well suited for the determination of the dynamic performance of any type of discrete coil guideway.

When the dynamic circuit theory is applied to the loop-shaped coil guideway, the model becomes relatively simple because (1) the currents or flux in the superconducting coils aboard the vehicle are usually fixed, (2) the voltages across the individual loop coils are zero, and (3) a connection transformation for the guideway coils is not needed. Assuming \( m \) superconducting coils moving above \( n \) loop-shaped guideway coils with a constant velocity \( v \), and neglecting vertical and horizontal perturbations, we obtain a system of voltage equations for the loop-shaped coil guideway from (2)

\[
\begin{bmatrix}
R_1 & R_2 & \cdots & R_n \\
L_{11} & L_{12} & \cdots & L_{1n} \\
L_{21} & L_{22} & \cdots & L_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
L_{n1} & \cdots & \cdots & L_{nn}
\end{bmatrix}
\begin{bmatrix}
i_1 \\
i_2 \\
i_3 \\
\vdots \\
i_n
\end{bmatrix}
= -v
\begin{bmatrix}
i_1 \\
i_2 \\
i_3 \\
\vdots \\
i_n
\end{bmatrix},
\]

where \( I_j (j = 1, m) \) are the currents in superconducting coils aboard the vehicle, \( L_{ij} (i, j = 1, n) \) is the mutual
inductance between the $i$th and $j$th loop coils on the guideway, and $R_i$ ($i = 1, n$) is the resistance of the $i$th loop coil. Both $L_{ij}$ and $R_i$ are constant if the loop coils are all identical. The term $G_{ij}(i = 1, n$ and $j = 1, m$) is the derivative with respect to $x$ of the mutual inductance between the $i$th loop coil on the guideway and the $j$th superconducting coil aboard the vehicle and is a function of space and time. It should be noted that the unknowns in (20) are the currents of the loop coils in the guideway. The formulas used to evaluate self-inductances and the mutual inductances can be obtained [10] and [13], and the derivative of the mutual inductances can be determined numerically from the mutual inductances. Equation (20) cannot be solved directly because matrix $[G]$, (20) can be solved as a set of linear algebraic equations for the unknowns $di_j/dt$ ($j = 1, n$), instead of having the linear differential equations solved for $i_j$. The currents at time $t$ are found by adding $(di_j/dt)At$ to the previous currents, as indicated by (21). Iterations between (20) and (21) will determine the currents in all loop coils as a function of time.

\[
\begin{bmatrix}
  i_1(t) \\
  i_2(t) \\
  \vdots \\
  i_n(t)
\end{bmatrix} = \begin{bmatrix}
  i_1(t - \Delta t) \\
  i_2(t - \Delta t) \\
  \vdots \\
  i_n(t - \Delta t)
\end{bmatrix} + \begin{bmatrix}
  di_1/dt \\
  di_2/dt \\
  \vdots \\
  di_n/dt
\end{bmatrix} \Delta t. \tag{21}
\]

The currents as a function of position are also obtained from the above results because of the relation $\Delta x = v_x \Delta t$. It should be noted that this approach is only a simple example. In fact, many numerical methods can produce more accurate results. After finding currents in the loop coils, we are able to determine the longitudinal, lateral guidance, and vertical suspension forces, $f_x$, $f_y$, and $f_z$, respectively, of the system as functions of time and displacement from (5) to (7):

\[
f_x = \sum_{i=1}^{n} \sum_{j=1}^{m} i_j G_{x,ij} l_j \tag{22}
\]

\[
f_y = \sum_{i=1}^{n} \sum_{j=1}^{m} i_j G_{y,ij} l_j \tag{23}
\]

\[
f_z = \sum_{i=1}^{n} \sum_{j=1}^{m} i_j G_{z,ij} l_j \tag{24}
\]

The three components of the magnetic force given by (22) to (24) include force pulsations, which depend on the geometry and the material characteristics of the loop coils. Time-averaged forces can be found from (22) to (24) by taking the time averages over any desired period. Since the mutual inductance between coils decreases rapidly as the distance between them increases, a good approximation is to consider only mutual inductances between a guideway coil and the first two or three coils on each side of it. Thus, (22) to (24) and all the mutual inductances and their derivative matrices discussed previously are simplified.

IV. APPLICATION TO THE FIGURE-EIGHT-SHAPED NULL-FUX COIL SUSPENSION

The folded figure-eight null flux coil EDS maglev concept was invented in 1969 [14]. The Japanese maglev group further developed the EDS maglev concept and invented several new null-flux suspension systems, including the side-wall planar null-flux coil suspension and guidance system as shown in Fig. 2, on which the Miyazaki test track was developed [15]-[18]. The major advantage of the null-flux coil suspension system is that it can provide very high lift-to-drag ratios, typically a value of several hundred, depending upon design and offset. The null-flux suspension can also provide very small magnetic drag at the null-flux equilibrium point, which is particularly helpful in starting a maglev vehicle from rest. The Japanese have succeeded in designing and testing several versions of an EDS maglev system based on this null-flux suspension concept.

The computation of magnetic forces for figure-eight-shaped null-flux suspensions on the basis of the field and harmonic analyses has been discussed in the literature [15]-[16]. In this section, we apply dynamic circuit theory to the figure-eight-shaped null-flux coil suspension system. One advantage of this approach is that it can predict transient and dynamic performance on the basis of a simple and direct solution. For general purposes, we assume that $n$ superconducting coils interact with $n$ figure-eight-shaped null-flux coils, as shown in Fig. 3, and that the null-flux coils make up 2n loops. Assuming the currents in the superconducting coils to be held constant and neglecting the speed voltage terms resulting from the motions in the y-z plane, we can write general voltage equations in matrix form for the 2n-loop system. Because the currents in the upper loops equal those in the lower loops (but have the opposite spatial orientation), the system has only $n$ unknown currents, and we can apply a connection transformation to the system as discussed previously. The transformation submatrix for the guideway coils, $T_g$, is determined from the current relations, $i_j = -i_{n+j}(j = 1, n)$ as

\[
T_g = \begin{bmatrix}
  1 & -1 & \cdots & -1 \\
  1 & 1 & \cdots & -1 \\
  \vdots & \vdots & \ddots & \vdots \\
  1 & 1 & \cdots & 1
\end{bmatrix}. \tag{25}
\]

Because the transformation is only applied to the guideway null-flux coils, the transformation submatrix for the vehicle coils $T_s$ is a unit matrix. By using equations discussed in section 2.2 and (25), we obtain the voltage equations for the null-flux coil system in matrix form after
where $i_j (j = 1, n)$ is the current in the $j$th null-flux coil, and the prime is omitted because the current in the $(n + j)$th loop system is equal to that in the $j$th loop. The term $I_j (j = 1, m)$ is the current in the $j$th superconducting coil. The individual elements in the coefficient matrices after transformation are given as follows:

$$R_j' = R_j + R_{n+j}$$
$$j = 1, n$$

$$L_{ij}' = (L_{ij} + L_{n+i,n+j}) - (L_{i,n+j} + L_{j,n+i})$$
$$i = 1, n \text{ and } j = 1, n$$

$$G_{ij}' = G_{ij} - G_{n+i,j}$$
$$i = 1, n \text{ and } j = 1, m$$

where the prime quantities in the left-hand sides represent the equivalent circuit parameters of the null-flux coil system. The term $R_j'$ is the resistance of the $j$th null-flux coil, $L_{ij}'$ is the mutual inductance between the $i$th and $j$th null-flux coils, and $G_{ij}'$ is the derivative of the mutual inductance between the $i$th null-flux coil and the $j$th superconducting coil. The right-hand side represents the parameters before transformation. Thus, $R_j$ and $R_{n+j}$ are the resistances in the upper and lower loops of the $j$th null-flux coil, respectively; $L_{ij}(i = 1, 2n$ and $j = 1, 2n)$ is the self- or mutual inductances between the individual loop coils; and $G_{ij}$ and $G_{n+i,j}$ are the derivatives of the mutual inductances between the upper and lower loop of the $i$th null-flux coil and the $j$th superconducting coil, respectively. If we assume all loops of the null-flux coil to be identical, (27) and (28) can be simplified as

$$R_j' = 2R_j = 2R$$

and

$$L_{ij}' = 2(L_{ij} - L_{i,n+j}).$$

Using (29) to (31), we can rewrite (26) in terms of the individual loop-coil parameters:
Several important points should be noted about (32). First, the currents induced in the null-flux coils are due to the speed voltages in the right-hand side of (32). The speed voltages are given by the product of the vehicle speed \(v, \) superconducting coil current \(I_j (j = 1, m), \) and the difference in the derivatives of the mutual inductance between the moving vehicle coils and the upper and lower loops of the null-flux guideway coils. This means that the suspension force depends upon the product of those three factors. Second, by comparing (32) with (20), one can conclude that the current induced in the null-flux guideway coils are much smaller than those in the loop guideway coils for given superconducting coil currents and vehicle speed. It is seen from (32) that for the best situation, assuming the superconducting coils to be far away from the null-flux equilibrium point (that is, \(G_l >> G_{n+i,j}\)), the current induced in the null-flux coil guideway is only about one-half of that in the loop-shaped coil guideway, because the speed voltage term in the right-hand side of (32) is about one-half of that in (20). From the viewpoint of the lumped electric circuit parameters, the resistance and the self-inductance in each null-flux coil are two times larger than those in a single loop coil. However, the current may be further reduced due to the reversed connection between the upper and lower loop coils. Both factors are observed in (32). Clearly, power losses in the null-flux coils are greatly reduced when compared with loop-shaped coils because of small induced currents. However, the null-flux suspension force may not necessarily be reduced, because the derivative of the mutual inductance between the superconducting coils and the null-flux coils (with respect to the lift force axis) can be much larger than that between the superconducting coils and the loop coils. This explains why the lift-to-drag ratio in the null-flux suspension system tends to be greater than that in the loop-shaped coil suspension system. Third, because the upper and lower loops of each null-flux coil are wound in the z-direction relative to the null-flux position \((z = 0),\) the restoring force (lift force for displacement below the \(z\) = 0 position) rapidly increases from zero to a maximum value. Hence, the vertical restoring force changes from zero to its maximum value for a displacement of one-quarter to one-third the height of one loop of the null-flux coil. On the other hand, the change in vertical force (lift force) for a single-loop coil guideway increases from zero at infinite \(z\) (effectively, two or three coil diameters) to a maximum at \(z = 0.\) Hence, the null-flux lift force is generally expected to be much stiffer than with the single-loop coil. The single-loop case effectively produces lift by flux compression, whereas the null-flux case produces lift by increasing the difference in flux linkage between the vehicle coil and the upper and lower loops of the null-flux coil.

The three components of the magnetic force—the longitudinal magnetic force \(f_x,\) the lateral guidance force \(f_y,\) and the null-flux lift \(f_z—\) are obtained from (5) to (7) and (29):

\[
f_x = \sum_{i=1}^{n} \sum_{j=1}^{m} i I_i \left[ \frac{\partial M_{ij}}{\partial x} - \frac{\partial M_{a+i,j}}{\partial x} \right],
\]

\[
f_y = \sum_{i=1}^{n} \sum_{j=1}^{m} i I_i \left[ \frac{\partial M_{ij}}{\partial y} - \frac{\partial M_{a+i,j}}{\partial y} \right],
\]

and

\[
f_z = \sum_{i=1}^{n} \sum_{j=1}^{m} i I_i \left[ \frac{\partial M_{ij}}{\partial z} - \frac{\partial M_{a+i,j}}{\partial z} \right].
\]

Note from (34) to (36) that all magnetic forces in the null-flux coil guideway are determined by the difference between the forces acting on the upper loop and the lower loop coils. All forces vanish at the null-flux equilibrium point, as expected.

V. TRANSIENT AND DYNAMIC PERFORMANCE OF THE NULL-FLUX COIL SUSPENSION

To demonstrate the capability of the model, the transient and dynamic performance of the null-flux coil suspension system was studied on the basis of the dynamic circuit model. A computer code for the null-flux coil suspension system was developed and validated by a small-scale experiment. Computer simulations were conducted for various cases, and the results are given in the following sections.

A. Superconducting Magnet Moving Past a Figure-Eight-Shaped Null-Flux Coil

To understand the transient and the dynamic performance of the null-flux coil suspension system, the computer simulations were performed first on a simple case in which a superconducting magnet (SCM) moves past a null-flux coil, as shown in Fig. 4. The dimensions of the SCM and the null-flux coil are given in Table I. The current induced in the null-flux coil and the suspension force and magnetic drag acting on the SCM were computed and plotted as a function of the center position of the SCM. Fig. 5 shows the dependence of the currents induced in the null-flux coil on the displacement of the SCM with the SCM speed as a parameter. It can be seen from Fig. 5 that at a low speed (say, \(5 \text{ m/s}\)), the current induced in the null-flux coil alternates its direction as the SCM moves across the null-flux coil. The current increases as the leading edge of the SCM approaches the null flux coil, and it decays and becomes negative as the trailing edge of the SCM departs from the left edge of the null-flux coil. Consequently, the null-flux lift force also varies from positive to negative at a low speed (5 m/s) as shown in Fig. 6, where the lift is positive from \(x = -1\) m to 0.4 m and becomes negative after \(x > 0.4\) m. In other words, the
Fig. 4. A SCM moves across a figure-eight-shaped null-flux coil.

TABLE I
DATA USED FOR THE COMPUTER SIMULATION OF A SINGLE SCM INTERACTING WITH A NULL-FLUX COIL

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Superconducting magnet</td>
<td></td>
</tr>
<tr>
<td>Length (x-direction)</td>
<td>1.7 m</td>
</tr>
<tr>
<td>Height (z-direction)</td>
<td>0.5 m</td>
</tr>
<tr>
<td>Current</td>
<td>100 kA</td>
</tr>
<tr>
<td>Ground null-flux coil</td>
<td></td>
</tr>
<tr>
<td>Length</td>
<td>0.55 m</td>
</tr>
<tr>
<td>Height/loop</td>
<td>0.31 m</td>
</tr>
<tr>
<td>Space between upper and lower loop</td>
<td>0.12 m</td>
</tr>
<tr>
<td>Number of turns/loop</td>
<td>36</td>
</tr>
<tr>
<td>Cross section (cm)</td>
<td>1 cm²</td>
</tr>
<tr>
<td>Aluminum conductor</td>
<td></td>
</tr>
<tr>
<td>Equivalent air gap</td>
<td>0.2 m</td>
</tr>
<tr>
<td>(center plane to center plane)</td>
<td>0.02 m</td>
</tr>
</tbody>
</table>

Fig. 5. Current induced in the null-flux coil as a function of SCM displacement with SCM speed as a parameter.

at a low speed (5 m/s), it contains two negative impulses as the SCM moves across the null-flux coil. However, the second impulse becomes positive as the speed increases to 139 m/s. To understand this behavior, it must be recognized that the longitudinal force component consists of a conservative and a dissipative part. The dissipative part always acts to slow the vehicle down and is obtained by integrating the longitudinal force curve over the traversal time or displacement. As shown in Fig. 7, the integral over the displacement contains both positive and negative contributions. These contributions tend to become more nearly equal as the speed increases, so that the net dissipative or negative impulse delivered to the vehicle becomes smaller with increasing speed. The positive contribution and the portion of the negative contribution exactly cancelled by the positive contribution constitute the conservative or nondissipative part of the longitudinal force. As the vehicle coil approaches the null-flux coil, it is pushed away from the null-flux coil (in the direction opposite to the direction of motion); that is, the vehicle coil receives a negative force impulse that increases the magnetic energy stored in the system. As the vehicle coil moves away from the null-flux loop, the vehicle coil is pushed away from the null-flux coil (in the direction of motion), decreasing the magnetic energy stored in the system.

The current induced in the null-flux coil depends upon two factors: the time constant of the null-flux coil and the speed of the SCM. The time constant of the null-flux coil determines how fast the circuit allows the current to build up and decay, and the speed of the SCM determines how fast the flux linking the null-flux coil can change. The SCM speed also determines the time required by the SCM to travel across the null-flux coil. Letting \( T \) be the time constant of the null-flux coil and \( r \) be the pole pitch of the SCM, one can define a characteristic ratio \( r \) as the time constant of the null-flux coil divided by the time required by the SCM to traverse one SCM pole-pitch distance:

\[
r = \frac{T}{r/v_x}.
\]
Fig. 7. "Magnetic drag" as a function of SCM displacement with SCM speed as a parameter.

Referring to Figs. 5 to 7, one obtains $r = 0.19$ for $v = 5$ m/s, $r = 1$ for $v = 28$ m/s, and $r = 5.2$ for $v = 139$ m/s. Numerically, as the characteristic ratio $r$ becomes larger, suspension performance improves. For $r > 1$ the current induced in the null-flux coil remains unidirectional during most of the traversal time, so that the lift force impulse is predominantly of one sign only, and the net dissipative force impulse is relatively small. Thus, the lift-to-drag ratio becomes larger. In general, one should design a coil guideway having a characteristic ratio $r$ larger than 1. The characteristic-ratio concept implies that one may improve the suspension performance near a maglev station, where the vehicle speed is usually low, by using more conductor to increase the guideway time constant and obtain a larger $r$. On the other hand, at high speed, the time constant of the guideway conductor can be made relatively small or the volume of the conductor used can be reduced.

B. A SCM Moving along a Null-Flux Coil Guideway

A computer simulation was performed on the second case where a SCM moves along a null-flux coil guideway at a constant speed (Fig. 2). This can also simulate the dynamic performance of a maglev vehicle being switched from one guideway to the next, where the null-flux coil of the second guideway is passive before the vehicle arrives. Figures 8 and 9 show the current induced in the first four coils of the guideway as a function of SCM displacement at 5 m/s and 139 m/s, respectively. As shown in Figs. 8 and 9, these currents are developed rapidly as the SCM approaches the coils, and the coupling between the two adjacent null-flux coils is relatively weak. This implies that one may be able to calculate magnetic forces on the basis of a model of one SCM interacting with one null-flux coil without causing significant error. Fig. 10 shows the dependence of the null-flux lift on SCM displacement, where one can see that at a low speed or a small value of the characteristic ratio $r$, the magnetic force is developed within one null-flux-coil length, and at a high speed or a large value of $r$, the magnetic force is developed within about three coil lengths. In other words, in the worst case, the electrical transient vanishes within three coil lengths. One can conclude that the electric transient does not significantly affect the magnetic force performance in a maglev system. However, the magnetic forces fluctuate as the SCM moves along the null-flux coil guideway. The amplitude of the fluctuation depends strongly upon the coil pitch of the null-flux coil. The coil pitch is defined as the length of the figure-eight coil plus the spacing between...
two neighbor coils. Figure 11 shows the dependence of the null-flux lift on the longitudinal position of the SCM with the distance between the null-flux coils as a parameter, where one can see the null-flux coil guideways having 12- and 21-cm coil spacings suffer from large force fluctuations, and those having 15- and 18-cm coil spacings have small force fluctuations. There seems to be an optimum spacing at around 18 cm that minimizes force fluctuations. However, the average force always increases as the spacing decreases. It is interesting to note from Figs. 10 and 11 that the frequency of the force fluctuation is given by the vehicle speed divided by the coil pitch of the null-flux coils. In Fig. 11, the length of the coil is 55 cm, and the vehicle velocity is 500 km/h for all four conditions. Thus, the frequencies of the force fluctuations are about 208 Hz for 12-cm spacing, 199 Hz for 15-cm spacing, 190 Hz for 18-cm spacing, and 183 Hz for 21-cm spacing.

C. Evaluation of the Side-Wall Null-Flux Coil Maglev System

The computer code was also applied to a maglev system using a side-wall null-flux suspension that has dimensions similar to those given in [17]. Fig. 12 shows the cross-sectional view of the system on which the computer simulations were based. The vehicle is assumed to weigh 17 metric tons and operate at a speed of 500 km/h. Furthermore, there are 12 SCMs aboard the vehicle, each having a current of 700 kA turns. The dimensions of the SCMs and the null-flux coils used in this system are assumed to be the same as those shown in Table I. A number of computations were conducted with various parameter ranges, including the three-dimensional magnetic forces as functions of air gap, vertical displacement, lateral displacement, and vehicle speed. In this example, the time-averaged magnetic forces are determined. Figures 13 to 15 illustrate the performance predictions for the null-flux suspension system under various conditions. The simulation results show that a maglev system using the figure-eight null-flux suspension has many potential advantages over other systems, including a high vertical stiffness and a very high lift-to-drag ratio.

Fig. 13 shows the dependence of the time-averaged null-flux lift and guidance forces per SCM and the lift-to-drag ratio on vehicle velocity. It can be seen from Fig. 13 that the null-flux levitation system can achieve very high lift-to-drag ratios, typically about 250 at 500 km/h with a vertical offset of 3 cm. However, the ratio drops sharply as the vertical offset increases. This implies that it is necessary for the null-flux suspension to operate at a small value of the vertical offset to have a major advantage over other systems and to achieve high system efficiency. The lift force at this normal operation point is about 15 kN/magnet, or 180 kN for a 12-magnet vehicle. This force is sufficient to lift a vehicle of 17 to 18 metric tons.

Fig. 13 also shows that the lift force approaches a maximum value of 42 kN per superconducting magnet SCM at about 10- to 12-cm maximum vertical offset or maximum vertical displacement. Beyond a 12-cm offset, the lift force decreases, meaning that the suspension system becomes unstable. The lateral guidance force of the system is much smaller than the lift force for vertical displacements of less than about 6 cm. Thus, in the range where the lift-to-drag ratio is large, the guidance force is relatively small. Fig. 14 illustrates force-speed characteristics for the null-flux suspension. As expected, it is similar to any other electrodynamic suspension system. Both lift and guidance forces increase as the speed increases. The lift-to-drag and guidance-to-drag ratios are proportional to the vehicle speed; however, their slopes are quite different. At a speed of 500 km/h, the lift-to-drag ratio
VI. CONCLUSIONS

Dynamic circuit theory, as applied to the maglev problems, treats all magnetic forces acting among components of a maglev system as they arise from changes in magnetic energy stored in that system. It is shown in this paper that mathematical models based upon this theory can be readily constructed for moving vehicle magnets interacting with stationary conductor arrays distributed on a guideway. Models of two types of guideway conductors were analyzed to demonstrate the utility and versatility of the approach. Very general expressions were given for the time-dependent magnetic forces between vehicle and guideway components. The model can be used to predict the electrical transient and dynamic performance of a maglev system.

Numerical results based on the mathematical model of a null-flux coil suspension system were given. It can be concluded that the dynamic circuit theory provides a powerful approach to analyzing complex and heretofore challenging problems associated with EDS maglev systems.

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REFERENCES


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