Approaches to Permanent Magnet Circuit Design

Herbert A. Leupold

Abstract - The advent of high-coercivity/high remanence magnet materials has greatly extended the range of applicability of permanent magnets so that many formerly impracticable devices are now viable. This paper discusses some of the more fruitful approaches to the use of these materials in magnetic circuit design and how application of comparatively few and simple techniques can result in a large number of useful devices. The approaches discussed are estimation of permeances, magnetic cladding, equivalent pole densities and current sheets, analytic application of Maxwell's equations, the magnetic moment rotation theorem and magnetic "mirrors". Some novel devices resulting from these applications are cited.

Introduction

Permanent magnet design can be rather difficult because solutions for fields of arbitrary configuration are rarely analytic. Fortunately, many of the most useful geometries possess sufficient symmetry to make a solution or an acceptable approximation to a solution analytic. For less tractable configurations, there are rougher approaches to obtaining initial approximations that form starting points for computer-aided iterative improvements. These approaches were simplified by the advent of the rigid \((J > B)\), high energy product materials because they provide constant magnetomotive force regardless of the circuits in which they are placed. Therefore, it is now possible to provide a surprisingly extensive variety of field distributions by use of only a few relatively simple computational tools and architectural concepts. This paper discusses some of the more fruitful of these and gives examples of some of the resulting structures.

Estimation of Permeances

An old technique, whose usefulness was limited before the availability of rigid permanent magnets, is an analogy with electrical circuit theory in which the analogues of the electromotive force \(V\), the current \(I\) and the conductance \(G\) are the magnetomotive force \(F\), the flux \(Q\) and the permeance \(P\), respectively \([1,2]\). We then have a magnetic Ohm's law, i.e. \(\Phi\) is equal to the product of \(P\) and \(F\) in analogy to the usual electric expression where \(I\) is equal to the product of \(G\) and \(V\).

The magnetic form is more difficult to use since flux paths are not clearly defined as are the wires that form the current paths in electrical circuits. However, the former can be estimated by a division of the space in and about the structure into paths bounded by cylindrical, spherical and planar surfaces. The permeances of such paths are calculated exactly when possible and with simple approximations when necessary. Examples of such paths with their formulae are shown in Fig. 1. Since the magnetomotive force of a rigid (squared looped) magnet is given by the product of the remanence and magnet length, we have the formal equivalent of an electric circuit whose conductances and driving EMFs are known and hence the \(Q\)'s can be calculated by Ohm's law and all the other algorithms of electrical circuit theory.

This procedure can be surprisingly accurate if total fluxes are determined because errors accrued in the permeance estimation tend to cancel each other. For detailed field calculations, it is less satisfactory but still constitutes a good basis for rough "back of envelope" calculations that at least provide a starting configuration which can be refined by a computer-aided iterative process. The method can also provide estimates of field magnitudes and leakage fluxes in feasibility studies.

Figure 2 shows the cross section of a cylindrically symmetric speaker-type magnet together with its computer generated flux plot. The estimation of permeances (EOP)
method, when applied to this structure yields a total flux of 21.8 \( \mu \text{Wb} \). The more precise finite element analysis by computer resulted in 21.2 \( \mu \text{Wb} \), a difference of less than 3%. Figure 2 attests to the surprising accuracy afforded by the computer results.

Crude approximations of the EOP method with regard to both the qualitative topology of the flux paths \( P_i \) and the quantitative locations of the flux branchings, junctures and centers as indicated by the capital letters.

**Cladding**

The EOP method suggests that when magnetic structures provide field to enclosed spaces, one should be able to eliminate flux leakage by reduction of the outer surface of the structure to an equipotential. This can be accomplished by a "cladding" of the flux-supply magnets with other magnets of orientation and dimensions tailored to confine the flux [2-5]. It is analogous to the placement of "bucking" batteries in branches of electrical circuits from which current is to be excluded. An example is shown in Fig. 3 in the form of an ordinary horseshoe magnet. It is desirable to confine all of the generated flux to the gap between the pole pieces. For confinement to occur in the electric circuit analogue, no current may flow through the branches parallel to \( G_w \). Such currents can be prevented, by placement in those branches, of "bucking" batteries, \( V_C \) of potential equal and opposite to that of the original supply battery. Analogously, radially oriented cladding magnets, \( F_C \) of equal and opposite

\[
\rho = -\nabla \cdot \vec{M}
\]
and the surface pole density by

\[ \sigma = \hat{n} \cdot \vec{M} \quad (2) \]

where \( \hat{n} \) is the unit vector normal to the surface where \( \sigma \) is being calculated.

Also equivalent would be an electric current distribution with current densities for volumes and surfaces given by

\[ \vec{j} = \nabla \times \vec{M} \quad (3) \]

and

\[ \vec{k} = \hat{n} \times \vec{M} \quad (4) \]

Pole densities can then be placed in Coulomb's law and integrated over all space to find the field at any point of interest. Similarly, current densities can be placed into the Law of Biot and Savart and integrated.

This method is of limited usefulness because the integrals obtained from it are rarely analytic. However, for certain high symmetry situations, it can be very convenient and fruitful. Examples are determination of the on-axis fields of radially and axially magnetized cylindrical annular rings as illustrated in Fig. 6. Such rings can be stacked axially so that the resulting sequence alternates between radially and axially oriented rings, as in Fig. 7A. This stack is an exceptionally efficient arrangement for the production of alternating axial fields for electron beam focusing in travelling wave tubes (TWT's).

This stacks can be made as much as one or two orders of magnitude lighter and smaller than a conventional stack that employs iron pole pieces sandwiched between opposed axially oriented rings, as in Fig. 7B[7].

\[ H(x) = \frac{( \pi J \rho x^2 )}{2} \left[ \frac{a^2 + (x-c)^2}{b^2 + (x-c)^2} \right] \]

\[ + \ln \left[ \frac{a + (b + |x-c|)}{a + (b - |x-c|)} \right] \left[ \frac{a + (b + |x+c|)}{a + (b - |x+c|)} \right] \]

Fig. 6 Calculation of on-axis field of a uniformly magnetized radially oriented magnet ring.

Fig. 7 (A) Compact travelling wave tube (TWT) stack composed of alternately arranged array of axially and radially magnetized rings. (B) Conventional TWT stack with sandwiched iron pole pieces.
Boundary Conditions on Components with Uniform Internal \( H, B \) and \( M \)

A method developed by Abele [8-12] affords the design of shells that supply and confine uniform transverse fields to cylindrical spaces of arbitrary cross sectional shape. A detailed description of this method can be found in references [8-12] and only a brief qualitative description is given here. First, the specified boundary of the cylindrical working space is approximated by line segments if it has curved portions and if not, used as-is to form the inner boundary of a magnetic shell whose outer border, also consisting of line segments, is to be determined. The correct assumption is then made that the resulting shell can be composed of irregular blocks over each of which \( H, B \) and \( M \) are uniform. Then by application of Maxwell's equations at all boundaries, invocation of the requirements that the field exterior to the structure be zero everywhere, and that the field in the interior working space be uniform and of specified magnitude and direction, a family of solutions of varying efficiency can be obtained. One of these can then be chosen on the basis of manufacturing convenience, mass efficiency or the architectural peculiarities of the application. Figure 8 illustrates a square cylinder designed for Magnetic Resonance Imaging (MRI).

**Rotation Theorem**

A simple computational tool is the rotation theorem [13] which states that if in any two-dimensional dipole distribution, all the dipoles are rotated through an angle \( \theta \), the field everywhere will retain the same magnitude but will be reoriented through an angle \( -\theta \). This is useful in determination of the best array about a cylindrical working space to provide a transverse field there. If the sides of the encompassing magnetic tubes are to be of similar form, as are the segments of a circular shell, the theorem can be used to find the magnetic orientation of each segment for coherent addition of their fields. The segments traversed by the diameter in the desired field direction should be magnetized in that direction. For each other segment to contribute the same magnitude in the same direction, the rotation theorem implies that its magnetization should be at an angle of \( 2\theta \) to the desired field where \( \theta \) is the angular coordinate of the segment.

Figure 9 shows the resulting structure which has a very high field to mass efficiency and is therefore often called a "magic" ring. In fact, it can attain fields of arbitrarily high strengths if one is willing to expend the necessary material. The field to mass dependence is logarithmic viz.

\[
B = B_R \ln\left(\frac{r_o}{r_i}\right)
\]

(5)

where \( r_o \) and \( r_i \) are the outer and inner radii, respectively. Therefore, the usual practical upper limit to the working flux density is about 2.0 Tesla with presently available materials. Higher remanences \( B_R \) would increase this limit proportionately. The "magic" ring also confines flux to its interior and is therefore amenable to close packing and placement in areas where stray fields would be a nuisance.
Structures with Three Dimensional Magnetization Configurations

The cross section of any of the structures described in the last section can be rotated around its magnetic axis to form solids of revolution of cylindrical symmetry. An important example is the "magic" sphere (Fig. 12). Such structures yield flux densities one third larger than the "magic" rings from which they are derived, i.e.

\[ B = \left( \frac{4}{3} \right) B \ln \left( \frac{r_e}{r_i} \right) \]  

Therefore, the "magic" spheres are even more field efficient than the "magic" rings but they do not confine their flux completely. They have an external field of a dipole centered in the sphere with a strength of

\[ D = \frac{\mu_0 MV}{3} \]

where \( V \) is the volume of the spherical shell and \( M \) is its magnetization.

Compensating Shells

A useful technique for the elimination of stray fields from structures with dipole moments is the inscription of the structure in a uniformly magnetized spherical shell with a dipole moment equal and opposite to that of the structure. Since a uniformly magnetized shell has no interior field, it will not alter the internal field produced by the inscribed structure and yet will cancel its external field. Figure 12B shows a "magic" sphere so compensated.

Magnetic Mirrors

Any magnetic structure with a plane of anti-mirror symmetry can be cut in two along that plane and the halves can be placed on slabs of a perfect passive ferromagnet (PFF) defined by a zero internal field [17]. Figure 13A shows a magic hemisphere so placed on an iron slab that approximates
a PPF. The anti-mirror image of the hemisphere in the iron takes the place of the missing half and produces the same cavity field as the parent sphere. An advantage of the "magic" igloo is that only half as many expensive, hard-to-manufacture pieces are needed as for the full sphere. Another

Fig. 13 "Magic" igloos made from "magic" hemispheres (A) Transverse to iron plate, (B) Parallel to superconducting plate.

is easier access through holes in an iron slab rather than through an expensive magnet which entails a sometimes undesirable field reversal upon passage through its wall.

A disadvantage is that "magic" igloos can produce fields no higher than the saturation magnetization of the PPF used. The highest is that of Permendur or 2.3 T, still a very high field for so small a structure.

A perfect diamagnetic slab \( B_{\text{internal}} = 0 \) could be placed along a plane of mirror symmetry after a cut along that plane to form a "magic" igloo with its basal plane parallel to the fields (Fig. 13B). Unfortunately, the only such diamagnets are superconductors, but these are limited by their comparatively small lower critical fields, \( H_{C1} \). Diamagnetic and ferromagnetic slabs can be used together to form an eighth sphere of the same field as the parent as in Fig. 14.

Fig. 14 One-eighth of a "magic" igloo.

ACKNOWLEDGEMENT

The author extends his appreciation and thanks to Dr. Ernest Potenziani II for his helpful suggestions and aid in preparation of this manuscript.

REFERENCES

[13] K. Halsch, Proceedings of the Fifth International Workshop on Rare Earth-Cobalt Permanent Magnets and Their Applications, p. 73, Roanoke, Virginia, USA, 7-10 June, 1991