Abstract—The latest NIST result from the comparison of the quantized Hall resistance (QHR) with the realization of the SI ohm obtained from the NIST calculable capacitor is reported. A small difference between the 1988 result and the present result has led to a re-evaluation of the sources and magnitudes of possible systematic errors.

I. INTRODUCTION

THE LAST REPORTED comparison of the quantized Hall resistance (QHR) and the realization of the SI ohm from the NIST calculable capacitor was presented at CPEM 1988 [1]–[3]. From the results of the NIST ohm realization and QHR measurement, the NIST value for the QHR corresponding to the \( i = 1 \) plateau, which is now called the von Klitzing constant, was \( R_{K} = 25812.80723(61) \Omega \) [2]. This agrees well with the conventional value for the von Klitzing constant adopted internationally in 1990.

Recent comparisons at NIST of the QHR with the SI ohm realization, however, show a small difference. To determine the cause of this difference, the sources of possible systematic errors in both the ac and dc parts of the experiment have been re-evaluated.

II. MEASUREMENTS

The comparison of the QHR with the realization of the ohm through the calculable capacitor is made through the measurement of a 1000 \( \Omega \) transportable resistor. The ac and dc parts of this measurement sequence are shown in Fig. 1. The 1000 \( \Omega \) resistor, R311, consists of nine 1000 \( \Omega \) Evanohm resistors connected in a series-parallel configuration and mounted in a sealed metal can filled with mineral oil [4]. Present measurements indicate that the relative difference between the value of the 1000 \( \Omega \) resistor derived from the calculable capacitor and the value derived from the QHR based on \( R_{K} = 25812.807 \Omega \) is approximately \( 5.1 \times 10^{-8} \). This difference, which has been consistent over a period of three years since 1993, implies a value of \( R_{K} \) that is \( 4.2 \times 10^{-8} R_{K} \) larger than the value reported in 1988. This change is more than the expected relative standard uncertainty of the difference (i.e., the estimated relative standard deviation of the difference).

III. AC MEASUREMENTS

The calculable capacitor system and the rest of the ac measurement sequence are the same as described in [1]. The following components of possible systematic error have been re-evaluated:

a) The alignment of the calculable capacitor electrodes and the Fabry-Perot interferometer was checked. The angle between the calculable capacitor axis and the optical axis of the interferometer was found to be 34 \( \mu \)rad. This angle corresponds to a relative correction of \( 6 \times 10^{-10} \) in the capacitance measurement and this correction was applied. The separation of the calculable capacitor electrodes was also measured and found to have remained constant since 1988 to 0.1 \( \mu m \), which is the uncertainty of our measurement. However, recent linearity tests of the calculable capacitor have dictated an increase in the relative uncertainty for geometrical
imperfections from $5 \times 10^{-9}$ which was assigned in 1974 [4] to $15 \times 10^{-9}$.

b) The transformer ratio of the four-terminal-pair bridge used in the $10:1$ step-ups from 10 pF to 100 pF and from 100 pF to 1000 pF was measured. The relative difference between present and previous measurements was less than $1 \times 10^{-9}$, The transformer ratio for the 100:1 resistance bridge was also checked and the relative difference was less than $1 \times 10^{-9}$. The transformer ratio for the calculable capacitor bridge was measured and found to have a relative difference of $5 \times 10^{-9}$ from its previous value. This correction was applied in the new calculable capacitor measurements.

c) The voltage dependence of the 10 pF capacitor, which is used at 4 V and 14 V in the calculable capacitor bridge and then at 200 V for the 10:1 step-up, is measured during the time the SI ohm determination is performed. In addition, the voltage dependencies of the 100 pF and 1000 pF capacitors between 20 V and 200 V were also measured and appropriate corrections applied.

d) Uncertainties resulting from nonlinearities and phase defects in the bridge adjustments of all bridges were checked and found to be within the allotted uncertainties given in Tables I and II. Comparisons with previous measurements [5] of the nonlinearities showed them to be relatively constant. A quick check of the magnitude of the bridge adjustments was made in the current series of measurements through a comparison with a newly constructed 4-terminal-pair bridge (see item g) for a 10:1 setup with 1000 pF and 100 pF capacitance standards [6]. A 0.17 pF capacitor was added in parallel on the low side to create a significant change in the bridge reading. The relative agreement of both bridges was $3 \times 10^{-9}$ for this setup, which was within the known stability of the 0.17 pF standard, indicating that the magnitude of the bridge adjustments is quite satisfactory.

e) Coaxial chokes [7] were used in the bridges in every step of the SI ohm determination. Choke corrections were measured for each choke and applied to the bridge measurements. The sum of relative choke errors for any of the bridges was not larger than $2 \times 10^{-9}$.

f) The relative standard uncertainty due to harmonics in the quadrature bridge (1000 pF to 100 kΩ) was measured and found to be less than $1 \times 10^{-9}$.

g) A new 4-terminal-pair bridge has been constructed at NIST [6]. This was compared with the present 4-terminal-pair bridge used in the SI determination. From these comparisons it was determined that there was a problem with the grounding of the auxiliary balances of the present four-terminal-pair bridge when used for the 100 pF to 1000 pF step-up. This was corrected, but a relative correction of $5 \times 10^{-9}$ had to be applied to the ratio of the 100 pF to 1000 pF capacitors used in the present measurements.

IV. DC MEASUREMENTS

DC resistance scaling at NIST is based on both Hamon device and cryogenic current comparator (CCC) measurements [8]. Before 1992, Hamon scaling and potentiometric measurements at a one-to-one ratio were used with the QHR [3]. Beginning in 1992, the CCC technique has been used to compare 100 Ω reference resistors against the QHR.

There have been eight measurements of the 1000 Ω transportable resistor R311 since 1992 also using CCC scaling. Scaling was done with 100 Ω references that were assigned values against the QHR on approximately the same mean dates as the R311 measurements. The 100 Ω resistors are well characterized, with predictable drift rates, and require negligible temperature and pressure corrections. Based on a linear fit to the eight values, R311 has drifted at a rate of $153 \mu \Omega / yr$ and the residuals from the fit range from $+4 \mu \Omega$ to $-4 \mu \Omega$.

The following list summarizes the systematic effects that have been evaluated for the QHR and the dc scaling measurements.

- **a)** The current-linkage (CCC ratio) error has been determined to be negligible in the three CCC devices used at NIST. Since integrators provide the feedback signals, the gain of the nanovolt detector need not be calibrated. The output signal gain of each CCC bridge was calibrated after every measurement.
b) Leakage resistance errors in the CCC ratio bridges have been evaluated using in-situ measurements [9]. A small correction of order $-1.5 \mu \Omega$ has been applied to the dc assignments for 1000 $\Omega$ transfer resistor R311 due to leakage at the terminals. The CCC system that was used to compare the 1000 $\Omega$ transfer resistor R311 against the 100 $\Omega$ reference resistors has been used to compare the 6453.2 $\Omega$ QHR plateau against the 100 $\Omega$ resistors, providing some cancellation of possible leakage errors.

c) The 10:1 ratio of the CCC bridge used for the 1000 $\Omega$ measurements was compared to the same ratio of two other CCC systems in 1992 and 1994. The 10:1 ratios of the three systems were the same to within a relative standard uncertainty of $5 \times 10^{-9}$, which is the combined standard uncertainty for all the systems together.

d) Between 1992 and 1996, three different QHR devices of metrological quality have been compared against the primary NIST standard device called GaAs-8. The median dates of measurements of different devices were typically separated by less than two weeks. Some devices were compared using both directions of magnetic field and two QHR plateaus. The measurements at 12 906.4 $\Omega$ were made using a different CCC system from that used for 6453.2 $\Omega$ and 1000 $\Omega$ measurements. No relative difference of more than $2 \times 10^{-9}$ has been detected in the value of any QHR standard.

e) Loading effects have been determined to be negligible in certain NIST CCC bridge comparisons against a bank of well-characterized 100 $\Omega$ resistors [10]. The resistor R311 and a second 1000 $\Omega$ resistor were compared using CCC measurements at power levels of 0.26 mW and 1.0 mW in the 1000 $\Omega$ resistors. A few days later, the two 1000 $\Omega$ resistors were compared to one another using a room-temperature bridge with a power level of 4 mW. These comparisons differed by only 1 $\mu \Omega$, and help to check that any loading error is small in 4 mW (2 V) measurements of resistor R311.

f) In 1992, a second QHR system was operating at NIST with device GaAs-7, which had been used as the NIST primary standard since Jan. 1, 1990. A potentiometric measurement system [3] provided QHR-based assignments to three 6453.2 $\Omega$ transfer standards using a 1:1 ratio. These transfer resistors were also given QHR-based assignments using the present QHR CCC measurement system. The two QHR assignments using the two different systems agreed for each transfer resistor to within the relative standard uncertainty of the potentiometric system which was approximately $8 \times 10^{-9}$. The average difference was approximately $1 \times 10^{-9}$.

g) Comparisons of the 10:1 and 64.532:1 ratios of one of the CCC bridges used in the measurements against Hamon devices are described in [8]. The comparisons agreed to within a relative standard uncertainty of $1 \times 10^{-8}$ which is the practical limit of accuracy when using Hamon transfer standards.

h) The CSIRO National Measurement Laboratory of Sydney, Australia, and NIST participated in a comparison of dc resistance standards in Aug. and Sept. 1995 using transportable 1 $\Omega$ and 100 $\Omega$ references. This international comparison has not yet been fully analyzed, however, preliminary results indicate that the relative difference between the CSIRO/NML assignments and the NIST assignments for the three 1 $\Omega$ resistors was $(4 \pm 4) \times 10^{-9}$. The relative difference at the 100 $\Omega$ level was $(8 \pm 8) \times 10^{-9}$. These comparisons indicate that the relative difference of the QHR assignments of the two laboratories is no more than $8 \times 10^{-9}$.

V. RESULTS

The new assignment of an SI value for the von Klitzing constant through the measurement of the 1000 $\Omega$ resistor R311 is

$$R_K = 25.812.8[1 + (0.322 \pm 0.024) \times 10^{-5}] \Omega$$

(mean date 12/26/94).

The previous assignment in May 1988 [2] was

$$R_K = 25.812.8[1 + (0.280 \pm 0.024) \times 10^{-6}] \Omega$$

(mean date 5/17/88).

The uncertainties are standard uncertainties (i.e., one standard deviation estimates) representing the root-sum-square of the appropriate relative standard uncertainties listed in Tables I–III. The uncertainties listed in Tables I and II are for the ac measurements and those listed in Table III are for the dc measurements. The new assignment for $R_K$ is $4.2 \times 10^{-8} R_K$ larger than the value assigned in 1988. From our investigations of the measurement systems, we have found some possible explanations for this difference.

New calculations of the effects of loading in 6453.2 $\Omega$ transfer standards [10] indicate that loading corrections for these resistors were underestimated in 1988 by approximately $5 \times 10^{-9}$. This change in the loading corrections would increase the 1988 determination of $R_K$ by $5 \times 10^{-9} R_K$, bringing it into slightly better agreement with the new NIST determination.

Changes in the ratio of the current transformer, which is part of the 100:1 resistance bridge, may also be a source.

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**TABLE III**

**RELATIVE STANDARD UNCERTAINTIES IN RELATING THE QHR TO THE TRANSPORTABLE 1000 $\Omega$ RESISTOR R311**

<table>
<thead>
<tr>
<th>Source of uncertainty</th>
<th>Relative standard uncertainty (i.e., estimated relative standard deviation)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type A standard uncertainties</td>
<td></td>
</tr>
<tr>
<td>Variability of repeated observations in scaling QHR to 100 $\Omega$</td>
<td>$2 \times 10^{-4}$</td>
</tr>
<tr>
<td>Variability of repeated observations in scaling 1000 $\Omega$ to 100 $\Omega$</td>
<td>$2 \times 10^{-4}$</td>
</tr>
<tr>
<td>Type B standard uncertainties</td>
<td></td>
</tr>
<tr>
<td>Temperature dependence of resistors</td>
<td>$1 \times 10^{-9}$</td>
</tr>
<tr>
<td>Loading effects of resistors</td>
<td>$3 \times 10^{-6}$</td>
</tr>
<tr>
<td>Pressure dependence of resistors</td>
<td>$1 \times 10^{-6}$</td>
</tr>
<tr>
<td>Leakage</td>
<td>$3 \times 10^{-7}$</td>
</tr>
<tr>
<td>CCC bridge</td>
<td>$3 \times 10^{-6}$</td>
</tr>
<tr>
<td>Non-ideal QHR device</td>
<td>$2 \times 10^{-6}$</td>
</tr>
<tr>
<td>Drift and failure to close</td>
<td>$3 \times 10^{-6}$</td>
</tr>
<tr>
<td>Relative standard uncertainty</td>
<td>$7 \times 10^{-6}$</td>
</tr>
</tbody>
</table>
of the difference. In June 1994, this ratio was measured and found to have a relative difference of $3 \times 10^{-8}$ from its previous value. This new ratio remained unchanged for a series of measurements from June 6, 1994 until Sept. 23, 1994. However, the next measurement in Dec. 1994 indicated that the ratio had returned to its original value and it has not changed since then. If a change of this amount and of the requisite sign had occurred during the May 1988 SI ohm determination, it could be the cause of the observed difference. This transformer ratio has a history of being extremely stable and had not changed significantly from the 1970’s. At the time of the SI determination in May 1988, there was no reason to believe that it had changed from previous measurements, so it was not measured during that period. However, when the SI ohm determination was repeated in Oct. 1988, the assignment of $R_{K}$ was found to have increased by $2.8 \times 10^{-8} R_{K}$. Since this measurement was not done with all the accompanying checks as had the measurement in May 1988, it was assumed that this number was not as reliable. However, this measurement in Oct. 1988 may have been an indication that the current transformer ratio had returned to its value of the 1970’s.

Indeed, if the Oct. 1988 measurements had been used and the correction for loading of the transfer resistors applied, the assignment of $R_{K}$ would be $33 \times 10^{-9} R_{K}$ larger and would be in much better agreement with the present assignment. The extensive investigations which have been performed on both the ac and dc parts of this experiment have revealed no other causes for the observed difference in the assignment of $R_{K}$ than those described above. Unfortunately, it is not possible to reach a definite understanding as to the origins of the difference by making measurements eight years later.

VI. Conclusions

Periodic comparisons of the QHR with the realization of the ohm through the calculable capacitor ensures that the ohm based on the quantum Hall effect is consistent with the SI ohm. Our recent comparisons of the QHR and the realization of the ohm have been extremely consistent over the past three years of measurement. This new assignment of the quantum Hall resistance is based on this series of measurements, whereas the 1988 value was based on a single measurement. It is because of this that the authors believe that this assignment of the von Klitzing constant is more reliable. The results of this comparison will provide data for the least-squares adjustment of fundamental constants in 1997. In particular, assuming $R_{K} = \hbar/e^2$, our result implies $\alpha^{-1} = 137.036 \pm 0.00003$, where $\alpha^{-1}$ is the inverse fine structure constant.

References


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