A Low Field Determination of the Proton Gyromagnetic Ratio in Water

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Abstract—We measure the proton gyromagnetic ratio in H$_2$O by the low field method, $\gamma'_p(\text{low})$. The result $\gamma'_p(\text{low}) = 2.6751376 \times 10^4$ $\text{s}^{-1} \text{T}^{-1}$, leads to a value of the fine structure constant of $\alpha = 137.0359840$ (0.037 ppm) and a value for the quantized Hall resistance in SI units of $R_H = 25.812.80460$ $\Omega$ (0.037 ppm). To achieve this result, we measured the dimensions of a 2.1-m solenoid with an accuracy of 0.04 $\mu$m, and then measured the NMR frequency of a water sample in the field of the solenoid.

I. INTRODUCTION

AFTER COMPLETING a measurement of $\gamma'_p(\text{low})$ in 1979 [1] we began building an entirely new apparatus to further improve our measurements and consequently test quantum electrodynamic (QED) theory more stringently. We now report the first results of this latest effort.

The low field method of measuring the proton gyro-magnetic ratio in H$_2$O, $\gamma'_p(\text{low})$, involves two experiments. (The prime indicates that a spherical sample of pure H$_2$O at a temperature of 25°C is used.) First, we measure the dimensions of a precision single-layer solenoid by an inductive technique in which the position of the current in the wire is located [1, 2]. In the second part, we measure the proton precession frequency $\omega_p'$ by standard NMR techniques. $\gamma'_p(\text{low})$ is then obtained from: $\gamma'_p(\text{low}) = \omega_p' / \xi f$, where $\xi$ is the coil constant equal to the magnetic flux density for unit current calculated from the measured dimensions and $f$ is the current in the solenoid. The motivation for improving the accuracy of $\gamma'_p(\text{low})$ comes from its important contribution to our knowledge of the values of the fundamental constants, particularly the fine structure constant $\alpha$, since

$$\alpha^{-1} = \left(\frac{\mu_0 \rho_p}{\mu_p} \right) \left(\frac{R_H}{2e/h} \right)_{\text{LAB}}^{1/3}$$

and, therefore, the quantized Hall resistance $R_H$, since

$$R_H = \left(\frac{\rho_p^2 c^3 (\mu_0 / \mu_p)}{16R_m \xi'_{\text{LAB}} \left[2e/h \right]_{\text{LAB}}^{1/3}}\right)^{1/3}$$

because of the current interest in adopting a value of $R_H$ as a representation of the ohm. In these equations $\mu_0$ is the permeability of free space, $c$ is the speed of light in vacuum, $\mu_0 / \mu_p$ is the magnetic moment of the proton in units of the Bohr magneton, and $R_m$ is the Rydberg constant for infinite mass. These quantities are known to a few parts in $10^8$ or better. Note that the three electrical constants $R_H$, $2e/h$, and $\gamma'_p$ must be measured in the same laboratory (LAB) units, and that there is a cube root dependence on the measured quantities. Moreover, (1) and (2) do not depend on any direct measurement of SI electrical units. Therefore, our more accurate value of $\gamma'_p$ will help test the quantum Hall theory and the ac Josephson effect theory, as well as help to test QED.

The principal uncertainty in our previous determination was caused by the measurement of the solenoid diameter. For this new experiment we constructed a 2.1-m long by 0.295-m diameter solenoid. This longer solenoid allows us to employ a compensation technique which eliminates the need for an accurate measurement of the mean solenoid diameter.

II. REDUCING THE SENSITIVITY TO DIAMETER

The magnetic field of an ideal infinite helical solenoid has two favorable properties: the field inside is uniform, and the magnetic field is independent of the diameter, as it depends only on the number of turns per unit length (sometimes called the pitch). For solenoids of finite length, we have developed a compensation technique which retains these properties of field uniformity and reduces sensitivity to coil diameter when measuring the field at the center of the solenoid. This is accomplished by having five current sources, each of which puts current into segments of the solenoid as shown in Fig. 1. Using a computer program, we found many useful configurations and chose the one that gives a field uniform to better than 0.2 ppm within a spherical volume 8 cm in diameter, with an insensitivity to solenoid diameter similar to that of a 1.5-km long solenoid. In effect, the extra current in the end windings compensates for the finite length of the...

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solenoid. The solenoid has to be long enough as otherwise it is difficult to find a useful solution using practical current levels. Therefore, we built this longer solenoid for our present experiment. This single layer solenoid is wound on a fused silica form, and the grooves in the form have been hand lapped in order to achieve high uniformity in the radius and pitch. The wire is gold-plated copper and has been hand lapped in order to achieve high uniformity and roundness. The solenoid pitch was drawn through a die directly onto the silica form to ensure wire uniformity and roundness. The solenoid pitch is 1.058 mm/turn and the wire diameter is 0.8 mm. Fig. 2 shows how uniformly it was constructed. With this solenoid and our compensation technique, we greatly reduce the need to measure the diameter. However, solenoid radius variations and pitch variations are still the critical dimensions that must be measured in order to calculate the magnetic flux density.

III. DIMENSIONAL MEASUREMENTS

The magnetic techniques used to measure the solenoid radius variations and the axial position (pitch) variations have been described in earlier papers [1]-[3], so we will just summarize the method here. An ac current with a special wave form [2], [3] is injected into the turns being measured (we measure ten at a time), and a voltage is induced into five coils wound on another fused silica form (see Fig. 3). Coils A, A', B, B' have about 600 turns each, while coil C has 300 turns, and the entire probe system can be moved along the solenoid axis. Coils A and A' are wired in opposition and form a linear differential transformer that has a sensitivity to axial displacements of 0.01 μm. As we move the current injector from one set of ten turns to the next, a laser interferometer measures the displacement of the pick-up coil assembly between successive null readings. This displacement is a measure of the distance between the centers of current of the measured turns. At the same time the other three coils are used to measure the radius variations. The voltage induced in coil C is inversely proportional to the radius of the activated turns, while the voltages B and B' are directly proportional. The number of turns in coil C is adjusted to cancel the sum of coils B and B'. Therefore, the total small voltage difference is very sensitive to radius changes, yet insensitive to axial position. This three-coil radius-to-voltage transducer has a sensitivity of 0.01 μm. We calibrate it by having extra turns at the end of the solenoid that have 50 μm greater diameter than the main solenoid. A percent error in this calibration produces a 0.016-ppm uncertainty in γ′. When measuring both the axial position and radius variations, it is necessary to measure the effect of horizontal and vertical displacements of the probe coils as well as changes in the probe's horizontal and vertical angle. The corrections that must be made for each of these departures from the axis have a quadratic dependence on the radius variations, so we position the probe at the peak of these curves and define this to be the center. Corrections are applied for motions off this center, but in all cases they are less than 0.3 μm and their uncertainties are at least ten times less. A bubble level is used to measure the departure from straightness as the current injector travels along the solenoid. The laser used was calibrated on a daily basis against an iodine stabilized laser [4].

Using these magnetic induction techniques we measured the solenoid radius variations and axial position variations as a function of turn number. Our technique measures the average position of ten turns, so the 2100
turns are measured as 210 positions. A pitch of 1.058 mm/turn has been subtracted from the axial measurements, and the result is plotted in Fig. 2. Fig. 4 shows the difference between one measurement set carried out on May 2-4, 1988, just after the NMR measurements and a set carried out on March 27-28, just before the NMR measurements. Only the critical region between ± 300 turns was measured on the later occasion. The error bars represent the standard deviation achieved in one data set.

IV. NMR Results

We have resolved a major systematic error that prevented us from reporting a preliminary result for $\gamma_1$ at the CPEM'86 meeting. We discovered and eliminated a time dependent leakage current that was caused by high voltage breakdown when we reversed the current through our solenoid. This leakage current passed directly between the solenoid terminals, so that it was difficult to detect even though it was large ($\sim 10^{-5}$ A).

We measure the NMR using a 3.5-cm diameter water sample. The 52-kHz frequency is detected by a tuned pickup coil having 800 turns wound in sections in a spherical arrangement. Helmholtz RF drive coils have 5 turns per section and are perpendicular to the detector coil. Both detector and drive coils are perpendicular to the solenoid field. A frequency synthesizer is used to sweep through the 0.015-Hz wide resonance. A lock-in amplifier detects both the in-phase and quadrature signals. The sensitivity of the result to the accuracy of setting the phase is 0.022 ppm/deg, and the phase can be adjusted to ±1.5 deg.

We used this NMR detector to measure the flux density gradient along the axis, but first we calculated the linear gradient produced by unbalancing the two 0.05-A current sources in Fig. 1. The solid curve in Fig. 5 is a sum of all predicted gradients along the axis while the double dashed curve is that predicted 1-cm off axis. The 3.5-cm sample averages the field over its volume, so the gradient measured via NMR (the $x$'s with error bars) agrees with the calculated gradient.

In testing for systematic effects we found that different detector coils produced different results. One coil was loaned to us by Vigoureux of NPL and another by Weyand of PTB, and we also used two NBS coils. To measure the correction for the susceptibility of the detector coil we designed another coil which was wound directly on the water sample like the PTB coil, but was small enough to fit inside the other coils. We then measured the NMR frequency with and without the coil under test. Table I shows that after correcting for the susceptibility of the detector coil, all coils give similar results. The susceptibility measured for the NBS no. 1 coil is in agreement with that measured for the 1979 experiment by another method.
We used the NBS no. 2 coil for the final data taking because its spherical shape gives a more uniform filling factor. We measured the critical 1-Q resistor against the NBS reference group just before and just after the NMR measurements, and the standard deviation of the measurements was 0.009 ppm. A 1.5-km long cable connecting the NMR and the Josephson voltage standard laboratories was used each day in between NMR measurements to calibrate our Zener voltage reference against 2e/h. One key feature of this cable is a coaxial guard system that has current flowing in the guard coaxial shield [5]. This guard reduces leakage by between 10-100 times, but even with the guard current off, the voltage transfer changed by only 0.01 ppm. Fig. 7 is a plot of the daily average of the NMR frequencies with error bars indicating the standard deviation of the day's average. The standard deviation of these daily averages is 0.052 ppm and represents one of the largest sources of error. We do not use the standard deviation of the mean because this is a daily average and we have not carried out systematic tests to higher accuracy. Although the long-term day-to-day scatter of the Zener diode reference does not affect the NMR observations, the short-term hourly scatter may be a significant contribution. We would like a better voltage reference. The temperature coefficient of the solenoid, 0.495 ppm/°C measured by NMR data and 0.496 ppm/°C by dimensional measurements, is consistent with the nominal value for the expansion of fused silica.

### V. Corrections for Iron

By moving an iron object closer to the solenoid during the NMR measurements and knowing that its systematic falls off as the 6th power of the distance, we can correct for the objects required near the experiment. The largest such correction (−0.0018 ppm) is required for the pump that circulates a fluorocarbon cooling fluid to a shower that controls the solenoid temperature. The iron in the ground around the nonmagnetic facility must also be accounted for. Fig. 8 shows a set of coils that were constructed to measure both the earth's iron content and the solenoid susceptibility. Current from the special waveform used in the dimensional measurements is passed through the inner and outer coils in opposite directions. Their turns ratio has been chosen such that a very small voltage is induced in the middle coil, but the two coils have as large a net dipole moment as possible. The image currents produced by this dipole in the earth can be detected as we move the three coil assembly closer or further from the earth. Fig. 9 is a log–log plot of the detected voltage as a function of distance above the earth just outside the room housing the solenoid. The line has the slope of three predicted by the cubic dependence in the distance from an infinite permeable sheet. By performing such measurements from 0.5 to 1.2 m underneath the solenoid we obtain a correction to γS of (−0.12 ± 0.03 ppm) for the solenoid position which is 2 m above the ground. Our solenoid is manufactured from fused sand, so iron impurities are possible although the manufacturer promised they would be small. In the center of the coil assembly used to measure the earth's susceptibility there is a sufficiently large flux density, and the hole is large enough to insert our 29-cm diameter solenoid. We, therefore, calibrated the assembly with a material of known susceptibility such as ethyl alcohol and measured the change off voltage induced in the detector coil as we placed a 10-cm section taken from the end of the solenoid inside the as-

<table>
<thead>
<tr>
<th>Coil name</th>
<th>Susceptibility Correction to B (ppm)</th>
<th>Difference from NBS No. 2 (ppm)</th>
<th>NMR difference (ppm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NBS No. 2</td>
<td>-0.033 ± 0.045</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>NBS No. 1</td>
<td>-0.066 ± 0.072</td>
<td>-0.035 ± 0.08</td>
<td>-0.080 ± 0.08</td>
</tr>
<tr>
<td>NPL</td>
<td>+0.273 ± 0.05</td>
<td>+0.306 ± 0.06</td>
<td>+0.322 ± 0.12</td>
</tr>
<tr>
<td>PTB</td>
<td>-0.472 ± 0.1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fig. 7. The NMR readings, averaged over each day. The error bars are the standard deviation of that day's measurements. The unweighted mean is 12.97 ± 0.032 ppm (σNMR = 0.016 ppm).

![Image of coil assembly](image1.png)

Fig. 8. The coil assembly used to determine the susceptibility of the Earth. Current in the inner and outer coils produces only a small flux in the center detector coil, which detects flux from currents in the Earth.

![Image of data plot](image2.png)

Fig. 9. A log–log plot of the voltage induced in the susceptibility detector coil versus height above the ground. The solid line has a slope of exactly 3 thus showing a cubic dependence on distance.
assembly. We also measured a section of the fused silica vacuum chamber. Fickett of NBS/Boulder also calibrated small core sections taken from another section. The measured susceptibilities all agree with the accepted susceptibility of pure fused silica. The correction calculated using this value of susceptibility is negligible because the five current configuration also makes this correction approach the zero correction of an infinite cylinder.

VI. RESULTS AND CONCLUSIONS

Table II summarizes most of the corrections that must be applied to calculate $\gamma_p$ and the corresponding estimates of uncertainty if appropriate. The dimensional measurements, the NMR measurements, and the various calibrations all contribute about 0.05 ppm each. Thus no one item is presently limiting the accuracy of the results. Our value for $\gamma_p$ is

$$\gamma_p(\text{low}) = 2.67513\,376 \times 10^8 \, \text{s}^{-1} \, T_{\text{NBS}}^{-1}.$$  

In another paper [6] we compute the following quantities that are derived from this work and compare them to other measurements:

from (1): $\alpha^{-1} = 137.0\,359\,840(51) \, (0.037 \, \text{ppm})$

from (2): $R_H = 25\,812.80\,460(95) \, (0.037 \, \text{ppm}).$

This value of $\alpha^{-1}$ agrees with the QED value, the difference being $(-0.054 \pm 0.038 \, \text{ppm})$, but differs by somewhat more than two combined standard deviations from the NBS absolute ohm realization, the difference being $(-0.102 \pm 0.043 \, \text{ppm})$. In Fig. 10 we have plotted our value of $\gamma_p$ along with the values that were considered in the 1986 adjustment of the fundamental physical constants [7]. The agreement with the precise QED value is satisfying, but the difference between our value and the NBS ohm value, which also has a relatively small uncertainty, is disconcerting. We plan to continue our measurements to test further for any errors. This result provides one of the most accurate routes for measuring $R_H$ in SI units and will help ensure that the value adopted for $R_H$ is near its SI value.

For convenient future reference, we express our value of $\gamma_p$ in terms of representations of the volt and ohm based on the following adopted values of the Josephson frequency to voltage quotient and the quantized Hall resistance:

$$R_H = 25\,812.807 \, \Omega$$

$$2e/h = 483\,597.9 \, \text{GHz/V}.$$  

These values are those likely to be adopted by the Comité Consultatif d'Électricité, CCE, for basing practical representations of the volt and ohm on the Josephson and quantum Hall effects. The result is

$$\gamma_p(\text{low}) = 2.67515\,427 \times 10^8 \, \text{s}^{-1} \, T^{-1} \, (0.11 \, \text{ppm}).$$

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REFERENCES


