A Reflection Coefficient Derivation for the
Q of a Reverberation Chamber

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Abstract—A reflection coefficient method is used to derive the quality factor (Q) of reverberation chambers of arbitrary shape. The results are applicable to walls of general materials, but reduce to the previous result for highly conducting walls with small skin depth. The reflection coefficient method is also used to derive the decay time of reverberation chambers.

I. INTRODUCTION

Reverberation chambers (also called mode-stirred chambers) are multimoded cavities that are used for radiated emissions or immunity measurements [1], [2]. They typically use mechanical stirring (paddle wheel) or frequency stirring [3], [4] to create a statistically uniform field. The quality factor (Q) is an important performance factor for reverberation chambers because it determines the field enhancement and the decay time [5], [6].

The purpose of this short paper is to present a new and more general derivation and expression for the Q of reverberation chambers. It is based on a reflection coefficient approximation rather than the usual skin depth approximation, but it reduces to the previous result [7], [8] when the walls are highly conducting with a small skin depth. The same reflection coefficient approximation is used to derive the decay time in Section III.

II. REFLECTION COEFFICIENT DERIVATION FOR Q

The geometry for an electrically large cavity (reverberation chamber) of volume V and surface area S is shown in Fig. 1. The interior region has free space permittivity ε₀ and permeability μ₀, and the walls have conductivity σw, permittivity εw, and permeability μw. Most reverberation chambers are rectangular boxes [2], but the shape here is arbitrary.

The cavity is assumed to be well stirred so that the power density Pd and the energy density W are uniform throughout V. Field uniformity in a mechanically stirred chamber has been studied experimentally with an array of electric field probes [2]. Power and energy densities are related by

\[ S_c = cW, \quad \text{where} \quad c = \frac{1}{(\varepsilon_0 \mu_0)^{1/2}}. \]  

(1)

The calculation of the wall loss is based on representing the electric and magnetic fields by an ensemble of plane waves that are uniformly distributed in angle [6], [8]. The power Pd dissipated in the walls can be written

\[ P_d = \frac{1}{2} S_c S (1 - |\Gamma|^2) \cos \theta. \]  

(2)

where \( \Gamma \) is the plane wave reflection coefficient, θ is the incidence angle shown in Fig. 1, and \( \langle \rangle \) indicates average over incidence angle. The stored energy U in the cavity can be written

\[ U = \frac{W V}{S_c V} = \frac{S_c V}{c}. \]  

(3)

The cavity Q can then be written

\[ Q = \frac{\omega U}{P_d} = \frac{2kV}{S(1 - |\Gamma|^2) \cos \theta}. \]  

(4)

where \( k = \omega/c \) and \( \omega \) is the angular frequency. Equation (4) is a general result for highly reflecting walls where \( 1 - |\Gamma|^2 \ll 1 \). The next step is the evaluation of the average value in the denominator of (4).

The reflection coefficients for horizontal (perpendicular) polarization \( \Gamma_h \) and vertical (parallel) polarization \( \Gamma_v \) are given by [9]

\[ \Gamma_h = \frac{\mu_0 k \cos \theta - \mu_0 (k_\perp^2 - k^2 \sin^2 \theta)^{1/2}}{\mu_0 k \cos \theta + \mu_0 (k_\perp^2 - k^2 \sin^2 \theta)^{1/2}} \]  

(5)

and

\[ \Gamma_v = \frac{\mu_0 k_\perp^2 \cos \theta - \mu_0 k_\perp^2 (k_\perp^2 - k^2 \sin^2 \theta)^{1/2}}{\mu_0 k_\perp^2 \cos \theta + \mu_0 k_\perp^2 (k_\perp^2 - k^2 \sin^2 \theta)^{1/2}} \]  

(6)

where \( k_w = \omega[\varepsilon_w (\varepsilon_w - j\sigma_w/\omega)]^{1/2} \). To account equally for both polarizations in (2) and (4), the average quantity can be written

\[ \langle (1 - |\Gamma|^2) \cos \theta \rangle = \langle 1 - \frac{1}{2} (|\Gamma_h|^2 + |\Gamma_v|^2) \rangle \cos \theta. \]  

(7)

The angular average in (7) can be written [6]

\[ \langle (1 - |\Gamma|^2) \cos \theta \rangle = \int_0^{2\pi} \left[ 1 - \frac{1}{2} (|\Gamma_h|^2 + |\Gamma_v|^2) \right] \cos \theta \sin \theta \, d\theta. \]  

(8)

For \( |k_w/\tilde{k}| \gg 1 \), the squares of the reflection coefficients can be approximated

\[ |\Gamma_h|^2 \approx 1 - \frac{4 \mu_0 k \text{Re}(k_w) \cos \theta}{\mu_0 |k_w|^2} \]  

(9)

and

\[ |\Gamma_v|^2 \approx 1 - \frac{4 \mu_0 k \text{Re}(k_w)}{\mu_0 |k_w|^2 \cos \theta}. \]  

(10)
This expression for $Q$ does not require that the walls are highly conducting. However, if the walls are highly conducting and $\sigma_w / (\omega \varepsilon_w) \gg 1$, then we have

$$Q \approx \frac{3V}{2\mu_0 \delta S}, \quad \text{where} \quad \delta \approx \frac{2}{(\omega \mu_w \sigma_w)^{1/2}}. \quad (12)$$

This is the usual expression for reverberation chamber $Q$; it has been derived from the skin depth approximation and an average over cavity modes for a rectangular box [7] or an ensemble of plane waves for arbitrarily shaped cavities [8]. In contrast, if $\sigma_w = 0$, then we have

$$Q \approx 3V \left[ \frac{\mu_0 \varepsilon_w}{\mu_0 \varepsilon_0} \right]^{1/2}. \quad (13)$$

This is the result for a wall with a large permittivity.

III. DECAY TIME

The transient problem can be treated in a similar manner. If the source is turned off at $t = 0$, then the change in cavity energy $U$ can be written [6]

$$dU = -P_d \, dt \quad (14)$$

where $P_d$ is given by (2). This differential equation has the following exponential solution

$$U \approx U_0 e^{-t/\tau}, \quad t > 0 \quad (15)$$

where

$$\tau \approx \frac{3V}{4\delta S \mu_0 \varepsilon_w} \quad (16)$$

and $U_0$ is the cavity energy at $t = 0$.

Further approximations can be made for the cavity decay time $\tau$ corresponding to $Q$ approximations in (12) and (13). For example, for highly conducting walls we have

$$\tau \approx \frac{3V}{2\mu_0 \sigma_w \delta}. \quad (17)$$

IV. CONCLUSION

The losses in a reverberation chamber have been analyzed by using a reflection coefficient approach. This approach is more general than the skin depth approach, but it reduces to the skin depth result for highly conducting walls. Both the cavity $Q$ and the decay time can be derived using this approach. This approach could be generalized to multilayered walls by using the reflection coefficients for layered media. A practical application of this extension would be an analysis of metal walls with a paint coating [2].