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$$\begin{aligned}\therefore p' &= [A\lambda + H\mu + G\nu - m(A + H + G)] / \Delta. \\ q' &= [H\lambda + B\mu + F\nu - m(H + B + F)] / \Delta. \\ t' &= [G\lambda + F\mu + C\nu - m(G + F + C)] / \Delta.\end{aligned}$$

(p, q, t) , (p', q', t') are the tangential co-ordinates of the join of the poles.

Let A', B', C' be the angles of the triangle of reference. The center is the pole of the line at infinity $a\sin A' + \beta\sin B' + \gamma\sin C' = 0$. The tangential co-ordinates of the center are obtained by substituting $\sin A', \sin B', \sin C'$ for λ, μ, ν in p, q, t and are

$$\begin{aligned}S_1 &= (A\sin A' + H\sin B' + G\sin C') / \Delta, \\ S_2 &= (H\sin A' + B\sin B' + F\sin C') / \Delta, \\ S_3 &= (G\sin A' + F\sin B' + C\sin C') / \Delta.\end{aligned}$$

\therefore The tangential equation of the center is $\lambda S_1 + \mu S_2 + \nu S_3 = 0$.

Write $a + ka'$ for a , $b + kb'$ for b , $c + kc'$ for c , $f + kf'$ for f , $g + kg'$ for g , $h + kh'$ for h in $A\lambda^2 + B\mu^2 + C\nu^2 + 2F\mu\nu + 2G\nu\lambda + 2H\lambda\mu = 0$.

Then the tangential equation of the four points of intersection of S and S' is $S + k\Phi + k^2 S' = 0$ where k is undetermined, and

$$\begin{aligned}\Phi &= (bc' + b'c - 2ff')\lambda^2 + (ca' + c'a - 2gg')\mu^2 + (ab' + a'b - 2hh')\nu^2 \\ &+ 2(g'h' + g'h - af' - a'f)\mu\nu + 2(hf' + h'f - bg' - b'g)\pi\lambda \\ &+ 2(fg' + f'g - ch' - c'h)\lambda\mu.\end{aligned}$$

The condition for equal roots for k is $\Phi^2 = 4SS'$, which is the equation of the four points of intersection.

186. Proposed by J. R. HITT; Professor of Mathematics, Coronal Institute, San Marcos, Texas.

If two sides of a triangle and its in-circle be given in position, the envelope of its circumcircle is a circle (*Mannheim*). [From Casey's *Sequel to Euclid*.]

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics in The Temple College, Philadelphia, Pa.

Let vertex A be origin, sides b, c the axes. Then $x^2 + 2xy\cos A + y^2 - bx - cy = 0$ is the equation to the circumcircle. Let this equation be written

$$D - bx - cy = 0 \dots (1).$$

Since the sides b, c and the inscribed circle are fixed in position, the tangents from A to the in-circle are constant.

$\therefore b + c - a = \text{a constant} = m \dots (2)$.

$a = \sqrt{(b^2 + c^2 - 2bc\cos A)}$. This in (2) gives after reduction,

$$m^2 + 2bc(1 + \cos A) - 2m(b + c) = 0 \dots (3).$$

c from (1) in (3) gives

$$2b^2x(1+\cos A)+2b[my-mx-D(1+\cos A)]+2Dm-m^2y=0.$$

The condition for equal roots of b is

$$2x(1+\cos A)(2Dm-m^2y)=[my-mx-D(1+\cos A)]^2$$

$$\text{or } [D(1+\cos A)-m(x+y)]^2=2m^2xy(1-\cos A)=4m^2xy\sin^2\frac{1}{2}A.$$

$$\therefore D(1+\cos A)-m(x+y)\pm 2m\sqrt{(xy)\sin^2\frac{1}{2}A}=0.$$

$$\therefore x^2+2xy\cos A+y^2-\frac{m}{2\cos^2\frac{1}{2}A}[x+y\pm 2(xy)\sin^2\frac{1}{2}A]=0.$$

$$\therefore x^2+2xy+y^2-4xys\sin^2\frac{1}{2}A-\frac{m}{2\cos^2\frac{1}{2}A}[x+y\pm 2\sqrt{(xy)\sin^2\frac{1}{2}A}]=0.$$

$$\therefore [x\pm 2\sqrt{(xy)\sin^2\frac{1}{2}A}+y][x\mp 2\sqrt{(xy)\sin^2\frac{1}{2}A}+y-\frac{m}{2\cos^2\frac{1}{2}A}]=0.$$

$$\therefore x\mp 2\sqrt{(xy)\sin^2\frac{1}{2}A}+y-\frac{m}{2\cos^2\frac{1}{2}A}=0,$$

$$\text{or } x^2+2xy\cos A+y^2+[m-4m(x+y)\cos^2\frac{1}{2}A]/4\cos^4\frac{1}{2}A=0.$$

This is the circle.

CALCULUS.

144. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Find the volume of the sphere, $x^2+y^2+z^2=2az$, (a) within the paraboloid $z=Ax^2+By^2$; (b) within the cone $z^2=Ax^2+By^2$.

Solution by the PROPOSER.

$$x^2+y^2+z^2=2az\dots(1), \quad z=Ax^2+By^2\dots(2), \quad z^2=Ax^2+By^2\dots(3).$$

$$\text{From (1), } z=a\pm\sqrt{a^2-x^2-y^2}=a\pm\sqrt{a^2-r^2}.$$

From $v=\int\int zrdrd\theta$ we get

$$v=4\int_0^{\frac{1}{2}\pi}\int_0^R\sqrt{a^2-r^2}d\theta rdr=\frac{4}{3}\int_0^{\frac{1}{2}\pi}[a^3-(a^2-r^2)^{\frac{3}{2}}]d\theta.$$

(a). From (1) and (2),

$$x^2+y^2+(Ax^2+By^2)^2=2a(Ax^2+By^2).$$

$$r^2+r^4(A\cos^2\theta+B\sin^2\theta)^2=2ar^2(A\cos^2\theta+B\sin^2\theta).$$